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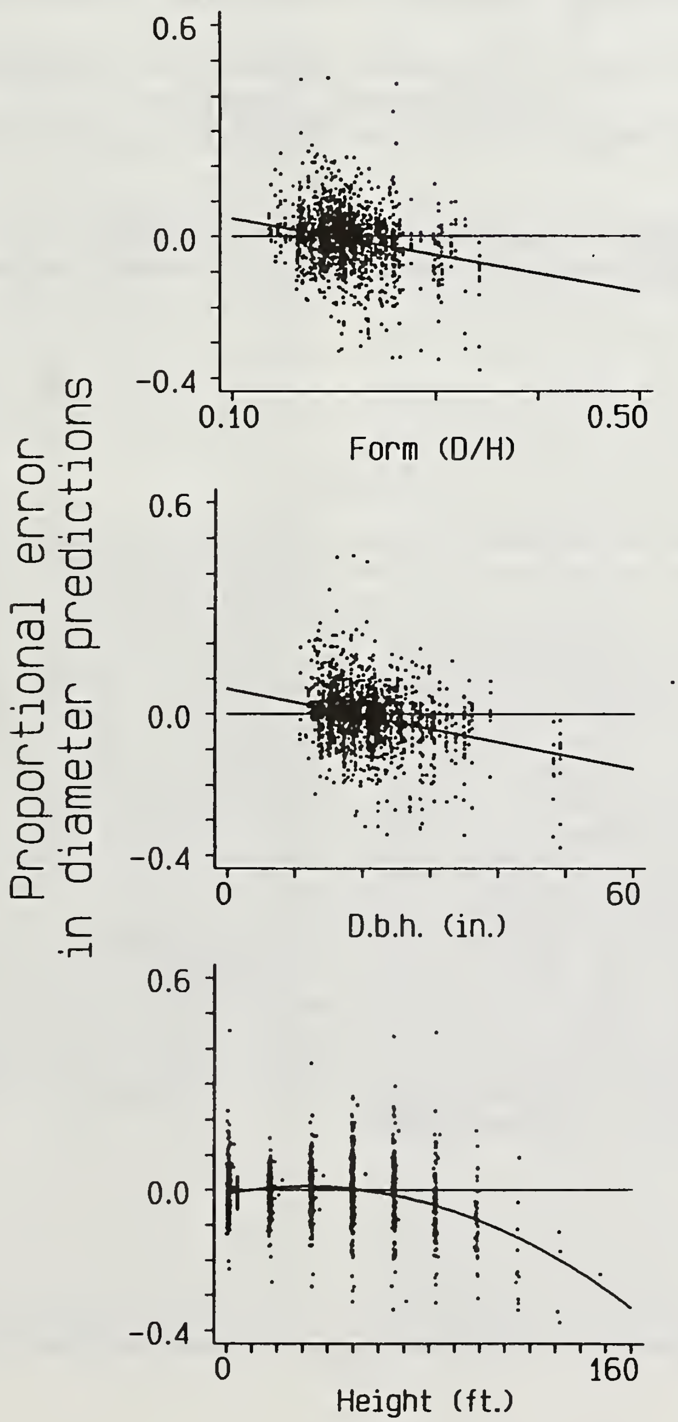
**Research Paper
RM-286**

Estimating Merchantable Tree Volume in Oregon and Washington Using Stem Profile Models

Raymond L. Czaplewski, Amy S. Brown, and Dale G. Guenther



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Estimating Merchantable Tree Volume in Oregon and Washington Using Stem Profile Models

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Abstract

Stem profile equations can estimate stem diameters using two standing tree measurements (or model predictions): diameter at breast height and total tree height. These diameter predictions are used in established scaling algorithms to estimate merchantable volume under different merchantability standards and scaling rules. This provides more flexibility than merchantable volume equations that must be revised whenever standards or scaling rules change. Estimates of merchantable wood volume are needed for timber sale preparation and administration, forest inventory, and strategic planning. The profile equation of Max and Burkhart was fit to eight tree species in the Pacific Northwest Region (Oregon and Washington), but overestimated stem diameters by an average of 0.06 to 0.24 inch. These biases were reduced using second-stage models that empirically correct for bias and weak patterns in the residuals. Most estimates of merchantable volume from the profile and second-stage models had an average error less than 10% when applied to independent test data for three national forests, and were usually within 6.5% of volume estimates from the hyperbolic profile equation of Behre, which has been used for decades in the Pacific Northwest; however, the Max and Burkhart equation does not require a standing tree measurement of Girard form class, as does the Behre equation.

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Management Implications

A stem profile model (i.e., taper model) predicts diameter at any height along the main stem using standing tree measurements of diameter at breast height (d.b.h.) and total tree height. Nominal log length, which specifies certain heights along the main stem, and minimum upper end diameter are common merchantability criteria. Given such criteria, a profile model can estimate the number of merchantable logs in each tree using a variety of merchantability standards. Log length and end diameters are also inputs to scaling algorithms that compute gross merchantable volume (i.e., unadjusted for defects) according to cubic or board foot scaling rules. Therefore, a single stem profile model can be used to predict volume using a wide variety of current and future merchantability standards, under different rules for scaling wood volume.

The Forest Service has used this approach for decades in the Pacific Northwest, with the Behre's hyperbola (Behre 1923) serving as the stem profile model. However, a new system of profile models was sought for two reasons. First, the Behre model, as applied in the Pacific Northwest, requires an estimate of diameter inside bark at the top of the first log (i.e., Girard form class), which is very difficult to measure in the field for standing trees, or accurately predict using more easily measured variables such as d.b.h. Biased estimates of Girard form class can produce unacceptably biased predictions of merchantable volume using the Behre model. Second, current data are available to fit profile models; these data better represent the population of trees currently being harvested.

Many stem profile models have been developed since the Behre model was first introduced (Sterba 1980). Based on the available literature, there is no one model that is consistently best for all estimated tree dimensions, all geographic regions, and all tree species. However, the segmented polynomial model of Max and Burkhart (1976) is consistently one of the better models. Cao et al. (1980) found that the Max and Burkhart model was the best predictor of upper stem diameters for loblolly pine (*Pinus taeda*) among the six models evaluated. Martin (1981) compared five stem profile models for Appalachian hardwoods. He recommends the Max and Burkhart model because it was the least biased and most precise predictor of diameters, height to a top diameter, and cubic volume, especially for the lower bole. Gordon (1983) found that the Max and Burkhart model is a less biased predictor of diameter than his compatible, fifth-order polynomial model. Amidon (1984) evaluated diameter predictions for eight stem profile models using five mixed-conifer species in California. The Max and Burkhart model was more precise and unbiased than most other models. The Alberta Forest Service (1987) evaluated 15 stem profile models for seven species groups. They recommend the Max and Burkhart model because of its generally superior ability to predict diameter, height, and cubic volume.

The only published comparison of profile models for the Pacific Northwest was done by Maguire and Hann (1987). They used modified versions of two stem profile models for predicting sapwood area at crown base for Douglas-fir (*Pseudotsuga menziesii*) from southwestern Oregon. The Max and Burkhart model was more effective than the alternative model, and consistently predicted more monotonic convex estimates of taper from breast height to crown base.

Data

Stem profile data for 46,402 sections from 7,615 trees were gathered by USDA Forest Service, Pacific Northwest Region (national forests in Washington and Oregon) from timber sales in the mid-1960's through the mid-1970's. Felled trees were selected for measurement, and measurements could be reconstructed from the stump to the tip. Diameter inside bark (d_{ib}), and associated bark thickness, were measured at stump height, breast height (4.5 feet), and the small ends of log segments that were nominally 16- or 32-foot lengths. Diameters were measured using a variety of techniques in the following order of preference: diameter tape and bark gauge, repeated caliper measurements and bark gauge, cross-sectional measurements and averaged bark thickness. Data from the following eight species were used: *Abies amabilis* (Pacific silver fir), *Abies grandis* (grand fir), *Abies magnifica* (Shasta fir), *Larix occidentalis* (western larch), *Pinus contorta* (lodgepole pine), *Pinus ponderosa* (ponderosa pine), Douglas-fir, and *Tsuga heterophylla* (western hemlock). Distribution of d.b.h. and total height for the 7,615 trees used to estimate model parameters by species and national forest (NF), are given in tables A1 through A8 in the appendix. In addition, 6,438 section measurements from 1,174 trees were used as independent reference data.

Stem Profile Model

The regression model used to estimate model parameters (i.e., b_i) for the Max and Burkhart model is

$$(d_{ib}/D)^2 = \left[\begin{array}{l} b_1(h/H-1) + b_2(h^2/H^2-1) + \\ b_3(a_1-h/H)^2 I_1 + b_4(a_2-h/H)^2 I_2 + \epsilon_1 \end{array} \right] \quad [1]$$

where

d_{ib} = upper stem diameter inside bark at height h
D = d.b.h. (inches) outside bark
h = height at the upper stem diameter prediction (feet)
H = total tree height (feet)
b_i = linear regression parameters (table 1)
a_i = join points (table 1); the upper join point is $i = 1$; lower is $i = 2$

$$I_i = \begin{Bmatrix} 1, \text{ if } h/H < a_i \\ 0, \text{ otherwise} \end{Bmatrix}$$

ϵ_1 = prediction error.

After parameters are estimated, the following prediction model is used to estimate diameter inside bark ($\hat{d}_1$):

$$\hat{d}_1 = D\sqrt{b_1(h/H-1)+b_2(h^2/H^2-1)+b_3(a_1-h/H)^2 I_1+b_4(a_2-h/H)^2 I_2} \quad [2]$$

The fitted models are shown graphically in figure 1, and estimated model parameters are in table 1. The join points (a_i) are nonlinear with respect to h/H, and were estimated using scatter plots (fig. 2) of the empirical first derivative of stem taper (Czaplewski 1989). Variance of the regression residuals, $(d_{ib}/D)^2 - (\hat{d}_1/D)^2$, was homogeneous within stem segments bounded by the join points, as judged by graphical analysis (Draper and Smith 1981) of scatter plots (fig. 1).

Bias in Predicting Inside Bark Diameter

The Max and Burkhart model produced biased estimates of inside bark diameters, which were overestimated by an average of 0.06 to 0.24 inch. These biases are large relative to their standard errors (table 2). The ratio of the mean to its standard error is the t-statistic, and is used to judge the relative magnitude of the bias. Because the usual assumptions of normality and independence required by t-tests are not reasonable here, we used t-values larger than 3 rather than the usual 2 as indicating the presence of bias.

Biased diameter predictions might be expected because the response variable in the regression model is $(d_{ib}/D)^2$, which is used to stabilize variance and reduce the nonlinear structure of the model. Transformation bias is made apparent when the prediction of $(\hat{d}_1/D)^2$ in equation [1] is subsequently retransformed to the prediction $\hat{d}_1$ from equation [2]. However, the magnitude of the transformation bias can be empirically estimated by a second-stage model. Less ad hoc methods are available to correct for transformation bias with the logarithmic transformation (e.g., Flewelling and Pienaar 1981) and power transformations (Taylor 1986). However, there are no directly analogous procedures available in the literature for the transformation $(d_{ib}/D)^2$ of d_{ib}.

Using the trees in our data set, the following second-stage model produced approximately unbiased predictions of diameter inside bark $\hat{d}_2$ using a biased prediction $\hat{d}_1$ from the Max and Burkhart model:

$$\hat{d}_2 = \hat{d}_1[c_1 + c_2 D + c_3(D/H) + c_4 h + c_5 h^2] \quad [3]$$

where c_1 to c_5 are regression parameters (table 2) and the expected value of $\hat{d}_2$ is d_{ib}. The bracketed term in equation [3] is normally very close to 1. The regression model used to estimate parameters in equation [3] is

$$(d_{ib}-\hat{d}_1)/\hat{d}_1 = (c_1-1) + c_2 D + c_3(D/H) + c_4 h + c_5 h^2 + \epsilon_2 \quad [4]$$

The form of this second-stage model was chosen using exploratory data analyses of the transformed residual errors $(d_{ib}-\hat{d}_1)/\hat{d}_1$ from the Max and Burkhart model. This transformation was selected to stabilize variance, avoid discontinuous changes at join points, produce a diameter estimate of zero at the top of the tree (i.e., $\hat{d}_2 = 0$ for $h = H$ because $\hat{d}_1 = 0$ for $h = H$), and permit parameter estimation using linear regression. Independent variables for

Table 1.—Regression statistics[a] and coefficients[b] for Max and Burkhart (1976) inside bark stem profile models.

Species	RMSE	b_1	b_2	b_3	b_4	a_1	a_2
Pacific silver fir	0.0892	-1.7420	0.6184	-0.8838	94.3683	0.50	0.06
Grand fir	0.1103	-1.5332	0.5600	-0.4781	129.9282	0.59	0.06
Shasta fir	0.0986	-2.5151	1.0729	-1.2346	172.8074	0.75	0.06
Western larch	0.1059	-1.3228	0.3905	-0.5355	115.6905	0.59	0.06
Lodgepole pine	0.1108	-1.2989	0.3693	0.2408	89.1781	0.41	0.06
Ponderosa pine	0.0934	-2.3261	0.9514	-1.0757	94.6991	0.72	0.06
Douglas-fir	0.0915	-2.8758	1.3458	-1.6264	20.1315	0.72	0.12
Western hemlock	0.0910	-2.0993	0.8635	-1.0260	91.5562	0.59	0.06

[a] *Root mean square error (RMSE) in* $(d/D)^2$ *units.* R^2 *statistics range from 0.97 to 0.98*
[b] *All coefficients are dimensionless, and are valid with both English and metric units. Join points (i.e.,* a_1 *and* a_2*) were estimated using the graphical techniques of Czaplewski (1989). The remaining coefficients (i.e.,* b_1 *to* b_4*) were subsequently estimated using multiple linear regression.*

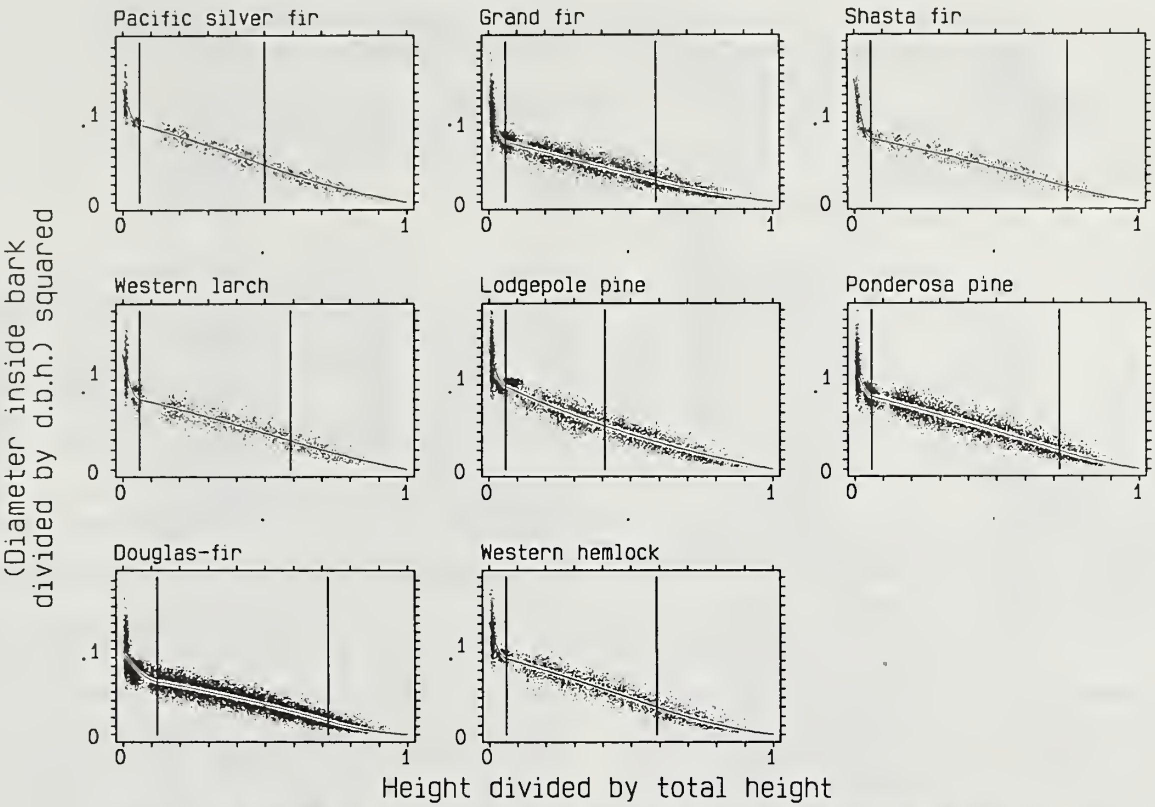

Figure 1.—Scatterplots of stem profile. Curves represent fitted Max and Burkhart model. Vertical lines represent join points (a_1 and a_2). Note that variance about the regression predictions is approximately homogeneous, at least within segments that are bounded by join points.

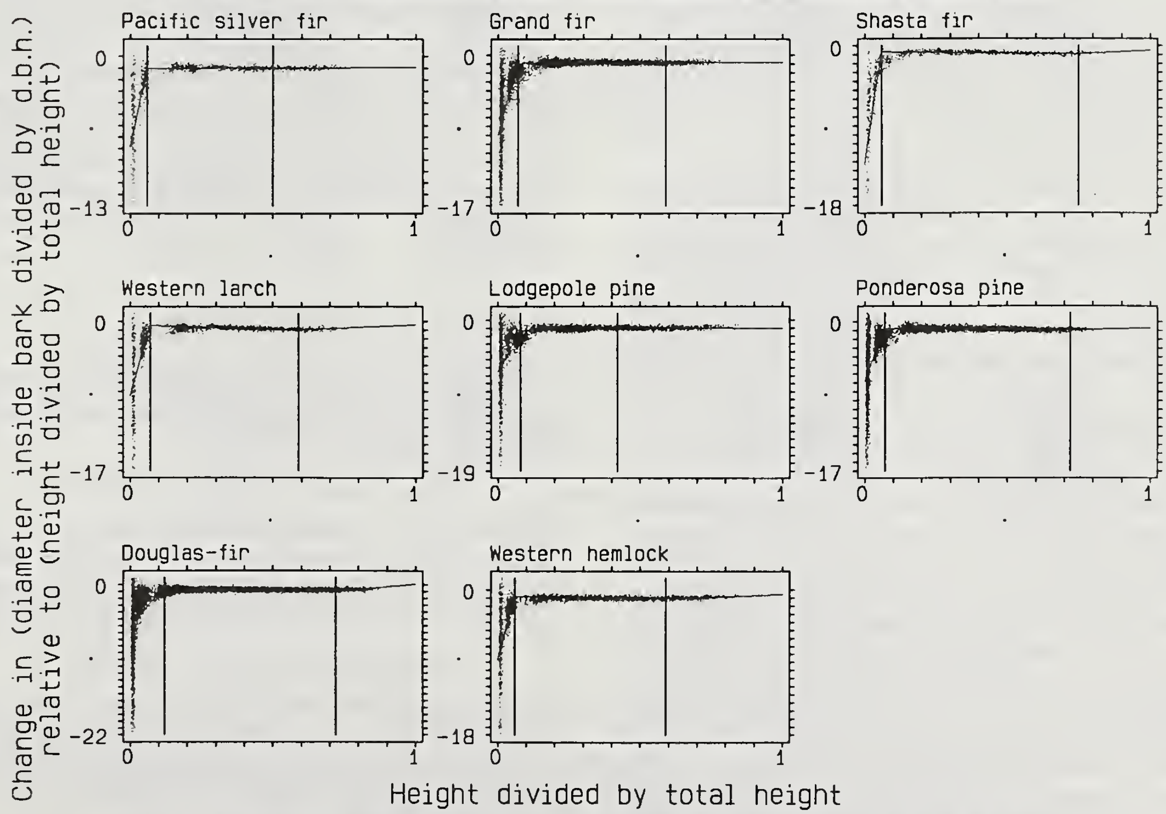

Figure 2.—Scatterplots of empirical first derivative (Czaplewski 1989) for change in stem diameter relative to height. These plots show changes in the rate of stem taper that correspond to join points (a_1 and a_2) in the Max and Burkhart model. Estimated join points are indicated by vertical lines. These estimates were used in table 1.

Table 2.—Stem profile and second-stage models for d_{ib} and a summary of error[a] in predicting upper stem diameters. *t*-statistics are used to portray the magnitude of bias relative to the standard error of the mean residuals.

	Mean residual error from Max and Burkhart model			Second-stage model for d_{ib}								
				Mean residual		Regression statistics[b]		Regression coefficients[c]				
	n	inches	*t*-stat.	inches	*t*-stat.	*F*-stat.	R^2	c_1	c_2	c_3	c_4	c_5
Pacific silver fir	1,735	-0.13	-4.63	0.01	0.38	69.96	0.11	1.046637	-0.0027747	0.0	0.0011203	-0.0000163
Grand fir	7,840	-0.21	-11.60	0.03	1.61	260.52	0.12	1.150112	-0.0017588	-0.4397838	0.0005402	-9.23e-06
Shasta fir	1,661	-0.13	-3.06	0.03	0.70	139.25	0.08	1.164438	0.0	-0.5995425	0.0	0.0
Western larch	2,224	-0.24	-6.94	0.03	1.04	137.98	0.16	1.106893	-0.0054016	0.0	0.0010691	-0.0000167
Lodgepole pine	5,877	-0.22	-11.37	0.01	0.69	193.34	0.09	1.080831	0.0	-0.4053328	0.0008174	-0.0000127
Ponderosa pine	10,194	-0.06	-3.79	0.02	1.37	41.07	0.01	1.032578	0.0011123	-0.2102077	0.0	-2.54e-06
Douglas-fir	13,318	-0.24	-13.99	0.01	0.41	224.56	0.06	1.101845	-0.0015528	-0.2367920	-0.0003635	4.25e-06
Western hemlock	3,553	-0.15	-5.57	0.01	0.53	65.06	0.03	1.073428	0.0	-0.3191318	0.0	-2.84e-06

[a]*Mean residual error is measured minus predicted* d_{ib} *(at stump, d.b.h., and end of merchantable 16-foot logs).* t-*statistic is the mean residual divided by its standard error of the mean (dimensionless).*

[b]*The degrees of freedom for the* F-*statistic are* (k-1, n-k-1) *where* k *is the number of non-zero regression coefficients (including* c_1*) and* n *is the number of measured sections.*

[c]*Coefficient units are:* c_1*, dimensionless;* c_2*, inches;* c_3*, inches/feet;* c_4*, feet;* c_5*, feet*^{2}*. Stepwise multiple linear regression was used to estimate* c_1 *to* c_5*. If one of these coefficients has a value of zero, then it indicates that it was eliminated during the stepwise process.*

predicting residuals from the Max and Burkhart model were selected using backward elimination stepwise regression. The transformed residual error, described in equation [4], was the response variable. The least significant independent variables in equation [4] were eliminated, one at a time, until the partial *F*-statistic for each regression parameter exceeded its critical value by a factor of 6, with $\alpha = 0.05$. This type of criterion is recommended by Draper and Smith (1981) to "distinguish statistically significant and worthwhile prediction equations from statistically significant equations of limited practical value." Regression parameters that were eliminated have values of zero in table 2.

Residual errors from the Max and Burkhart model were weakly correlated with the following independent variables, as illustrated in figure 3: tree size (represented by *D*), an index of tree form (measured by *D*/*H*), and a curvilinear function of height (*h*) of the upper stem diameter prediction. The practical significance of these correlations with independent variables varied by tree species. The second-stage model reduces the bias in predicted diameter, but the standard deviations of the residuals for diameter predictions decreased only 3%.

The effect of the second-stage model on the representation of tree form is not obvious. It is conceivable that the second-stage model produces illogical predictions of stem shape, especially for trees near extremes in the distribution of size and form. Therefore, the second-stage predictions of inside bark diameter ($\hat{d}_2$) were closely scrutinized for all trees in the data base. The multiplier that empirically corrects for bias, i.e., the bracketed portion in equation [3], does not greatly change the diameter estimate from the stem profile model. For all trees in this study, the extreme values were 0.71 and 1.10; 90% of the values were between 0.95 and 1.04, and half the values ranged between 0.98 and 1.01. The magnitude of these correction multipliers is similar to those found by Flewelling and Pienaar (1981) to correct for transformation bias in logarithmic volume equations. In addition, the second-stage diameter prediction ($\hat{d}_2$) is always positive and becomes smaller as height *h* increases for each tree. There were no unusual changes in the rate of stem taper for any tree, as evaluated using the second derivative of $\hat{d}_2$ with respect to *h* (Czaplewski 1989); there were no stair-step patterns, i.e., sign reversals in the second derivative as *h* increased for any one tree. Therefore, the second-stage model produced logical diameter estimates for all trees. However, it is possible that the second-stage model could poorly represent the form of a tree if its d.b.h. and total height are not represented by the trees in the data base (see tables A1 through A7), and this potential, although unlikely problem should be considered when the second-stage model is applied.

Predicting Diameter Outside Bark

The unbiased estimate of inside bark diameter $\hat{d}_2$ at height *h* is used to predict diameter outside bark $\hat{d}_{ob}$ at the same height:

$$\hat{d}_{ob} = \hat{d}_2 \{1 + \exp[c_6 + c_7(H-h)]\} \qquad [5]$$

where c_6 and c_7 are regression parameters. Scatterplots of the ratio $d_{ob}/\hat{d}_2$, and the model for $\hat{d}_{ob}/\hat{d}_2$ (equation [5]), are shown in figure 4. Regression parameters were estimated using the following regression model:

$$\log(d_{ob}/\hat{d}_2 - 1) = c_6 + c_7(H-h) + \epsilon_3 \qquad [6]$$

Scatterplots of this transformation, and plots of the linear regression models (equation [6]), are given in figure 5. Estimated regression parameters and statistics are in table 3; the mean error for estimates of d_{ob} are within 0.63 inch of zero.

This model represents a function of bark thickness, and was chosen based on exploratory data analysis of the

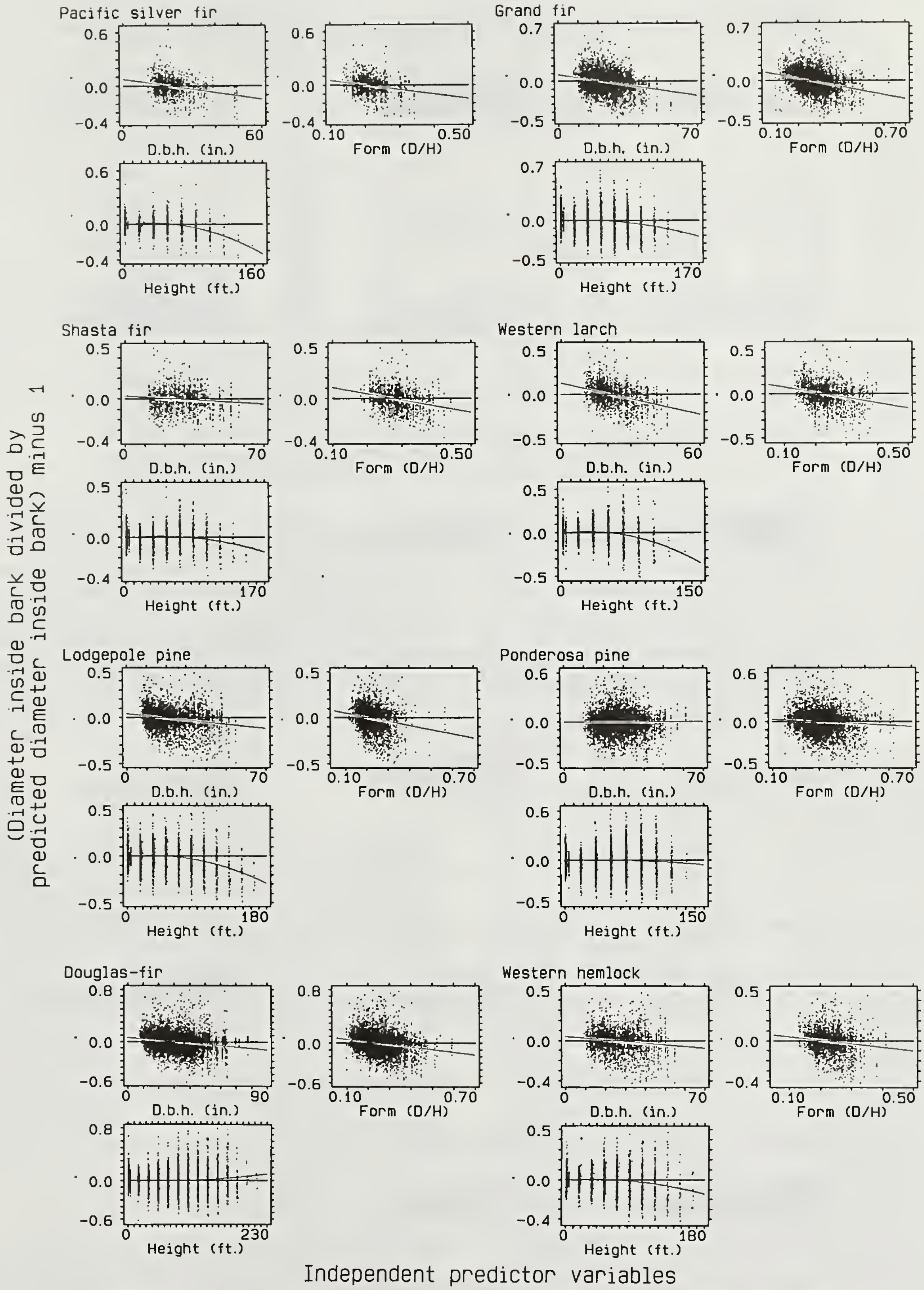

Figure 3.—Scatterplots of residual error in predicting diameter inside bark, using the Max and Burkhart model. The following independent variables are included: d.b.h. (*D*); an index of tree form, defined as d.b.h. divided by total tree height (*D/H*); and height of the diameter measurement (*h*). There are weak correlations of residuals with independent variables, which means that the stem profile model does not explain all patterns in the data. Regression models that predict residual error are given. These models use only one independent variable. Stepwise multiple regression models were fit to these residuals (table 2).

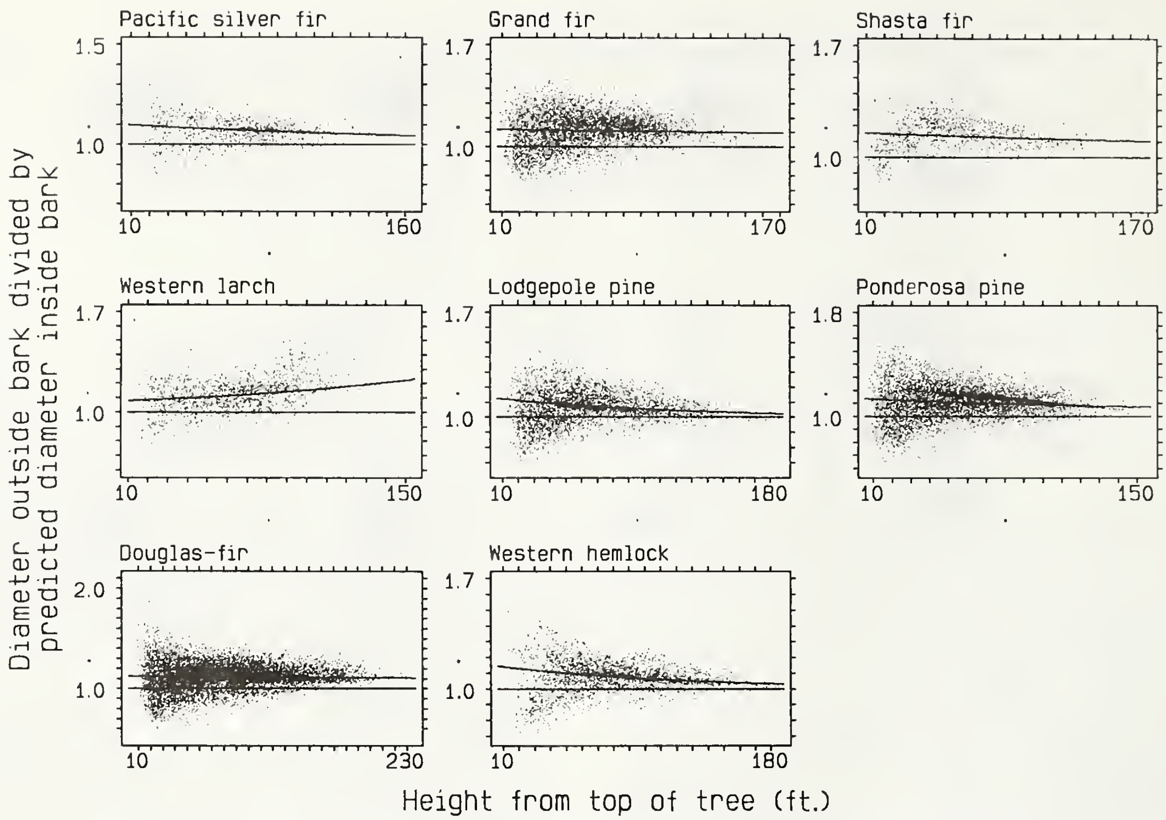

Figure 4.—Scatterplot of ratio between diameter outside bark (d_{ob}) and the unbiased prediction of diameter inside bark ($\tilde{d}_2$). Curves are fitted models, retransformed from the linear regression models that use log[($d_{ob}/\tilde{d}_2$) - 1].

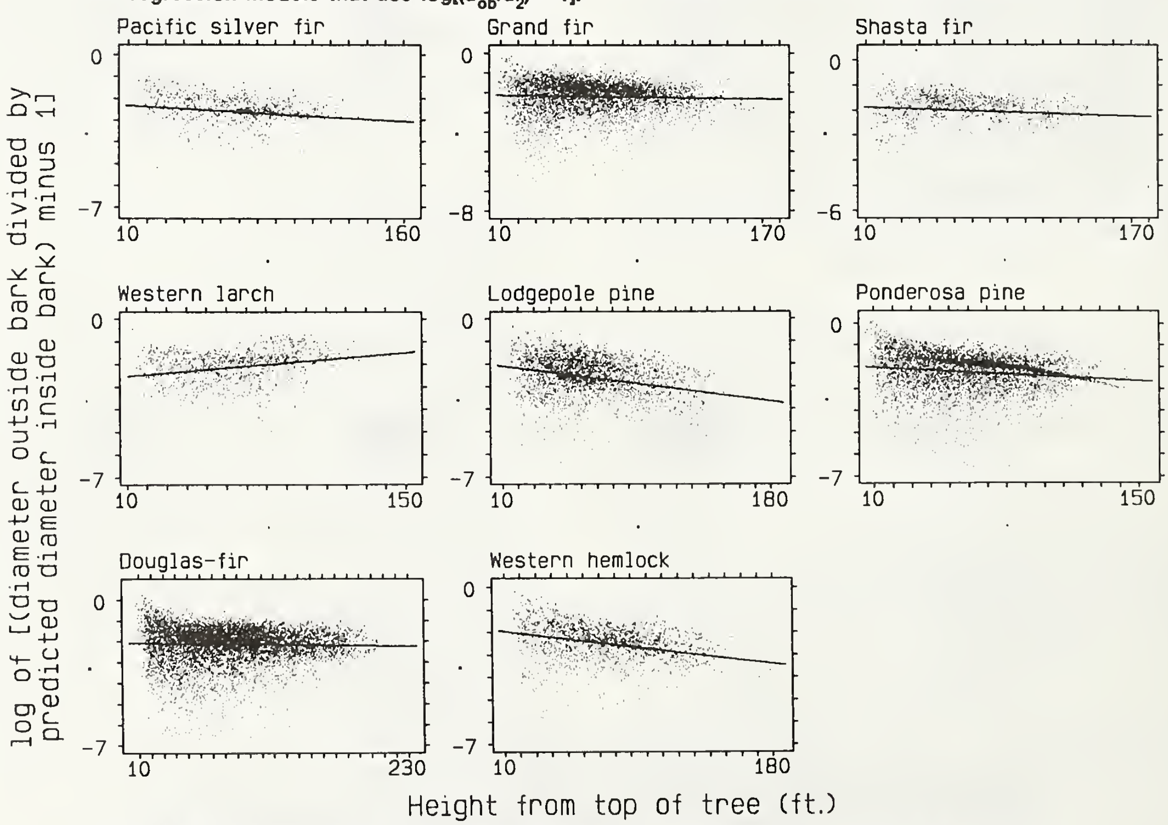

Figure 5.—Scatterplots of transformed variable, i.e., log ($d_{ob}/\tilde{d}_2$ - 1), used to fit model for diameter outside bark, given an unbiased prediction of diameter inside bark. Fitted regression line is also portrayed.

Table 3.—Second-stage models for d_{ob} (equation [5]) and a summary of errors[a] in predicting upper stem outside bark diameters.

Species	n	Mean residual		Regression statistics[b]		Regression coefficients[c]	
		inches	t-stat.	F	R^2	c_6	c_7
Pacific silver fir	1,735	-0.01	0.40	41.77	0.03	-2.277948	-0.0049568
Grand fir	7,840	0.15	7.44	11.20	0.01	-2.119883	-0.0012616
Shasta fir	1,661	0.25	5.62	18.80	0.01	-1.856899	-0.0023925
Western larch	2,224	0.20	4.20	97.92	0.05	-2.609802	0.0074167
Lodgepole pine	5,877	-0.02	-1.16	385.15	0.08	-2.015476	-0.0091009
Ponderosa pine	10,194	0.06	5.03	180.30	0.02	-1.987724	-0.0043569
Douglas-fir	13,318	0.63	90.45	270.62	0.02	-1.740004	-0.0029642
Western hemlock	3,553	-0.01	-0.25	264.18	0.08	-1.931856	-0.0073768

[a]*Mean residual error is measured minus predicted* d_{ob} *(at stump, d.b.h., and end of merchantable 16-foot logs).* t-statistic *is the mean residual divided by its standard error of the mean (dimensionless).*

[b]*The degrees of freedom for the* F-statistic *are* (k-1, n-k-1) *where* k *is the number of non-zero regression coefficients (including* c_1*), and* n *is the number of observations.*

[c]*Coefficient are dimensionless and were fit using linear regression with a logarithmic transformation.*

response variable ($d_{ob}/\hat{d}_2$). This model form was selected for four reasons. First, the variance of the response variable is homogeneous relative to h (fig. 5). Second, the regression parameters c_6 and c_7 could be estimated using linear regression and a logarithmic transformation. Third, the model always predicts a diameter outside bark that is larger than the estimated inside bark diameter because $\exp[c_6 + c_7(H\text{-}h)]>0$. Fourth, the estimated tip diameter is zero ($\hat{d}_{ob} = 0$ for $h = H$) because $\hat{d}_2 = 0$ for $h = H$.

Independent Test

The preceding results describe fit of the Max and Burkhart and second-stage model diameters to the same data that were used to build the models. In the following test, these regional profile and second-stage models are applied to independent validation data from three national forests (i.e., a subregion of the entire study area). The validation data were not used to fit the Max and Burkhart and second-stage models. The accuracy of predicting merchantable volume, in addition to stem diameters, is evaluated.

Accuracy of the Max and Burkhart and second-stage models is also compared with the Behre profile model that is presently used in the Pacific Northwest, as given by Bruce (1972). The Behre profile model is

$$\hat{d}_{ib} = \frac{D(F/100)[1-(h-h_f)/(h-h_f)]}{1 + (c-1)(h-h_f)/(H-h_f)}$$

where F is Girard form class, which is defined as the diameter inside bark at the top end of the first 16- or 32-foot log expressed as a percentage of d.b.h., h_f is the height aboveground of the top of the first log, and c is a constant. Diameters below h_f are not defined by the Behre model; however, a separate regression model has been fit by the Pacific Northwest Region to estimate stump diameter at 1 foot using d.b.h.

A new model for predicting Girard form class was developed using the validation data. Therefore, comparisons between the Max and Burkhart and Behre models are confounded by the lack of independence between the validation data and predictions from the Behre models; the accuracy of the Behre model is somewhat exaggerated. However, this comparison is presented as the best possible.

Validation Data

Data were gathered for the Deschutes National Forest (NF) in 1987, and the Mt. Hood and Siuslaw NF's in 1988. The species sampled in the Deschutes NF were Shasta fir, grand fir, lodgepole pine, ponderosa pine, Douglas-fir, and western hemlock. On the Mt. Hood NF, Pacific silver fir, grand fir, western larch, lodgepole pine, ponderosa pine, Douglas-fir, and western hemlock were sampled. On the Siuslaw NF, two species were sampled: Douglas-fir and western hemlock. Additional species in each of the forests were sampled, but had small sample sizes, or were species that had no profile model; these were not used. The range, mean, and standard deviation for d.b.h. and total tree height are given for each species in the three forests in table 4.

A Tele-relaskop[2] was used to make standing tree measurements of d.b.h. (D_s), upper stem diameters (d_s), and total tree height (H). Sightings were made at a variety of compass bearings. D.b.h. (D) was also measured with a diameter tape. Diameters were measured at heights that correspond to nominal 16- or 32-foot logs. Most Tele-relaskop diameter measurements at breast height were within 3% of the tape measurements. The tape and Tele-relaskop measurements of d.b.h. were compared (i.e., D and D_s), and trees with a difference larger then 10% (i.e.,

[2]*The use of trade and company names is for the benefit of the reader; such use does not constitute an official endorsement or approval of any service or product by the U.S. Department of Agriculture to the exclusion of others that may be suitable.*

Table 4.—Means, standard deviations (SD), range, and number of trees measured (n) in the independent test data, grouped by d.b.h. and total tree height by species and forest.

National forest species	No. trees	No. sections	D.b.h. (feet)				Total tree height (inches)			
			Mean	SD	Min.	Max.	Mean	SD	Min.	Max.
Deschutes										
Grand fir	28	86	15.4	3.1	8.8	21.1	79	19	31	107
Shasta fir	18	58	16.8	5.0	9.6	26.6	75	24	43	125
Lodgepole pine	76	202	11.4	3.5	5.5	23.7	60	17	27	110
Ponderosa pine	89	261	17.6	6.7	6.3	35.7	69	25	24	142
Douglas-fir	35	120	16.7	5.0	7.2	29.3	88	21	53	130
Western hemlock	14	54	16.9	10.2	8.1	47.1	96	31	50	176
Mt. Hood										
Pacific silver fir	70	263	17.1	8.1	5.0	37.5	85	35	28	164
Grand fir	77	316	18.3	6.7	7.0	34.9	95	24	44	152
Western larch	28	133	19.6	6.6	9.5	34.3	107	25	64	153
Lodgepole pine	44	166	12.1	4.4	5.9	28.1	70	23	37	142
Ponderosa pine	78	368	24.8	10.3	6.8	44.9	107	39	34	173
Douglas-fir	215	1,139	26.3	13.9	5.9	71.4	125	44	37	226
Western hemlock	139	643	22.0	9.6	5.9	45.4	111	38	37	271
Siuslaw										
Douglas-fir	177	1,770	29.6	12.7	6.7	67.5	144	42	48	218
Western hemlock	86	859	19.8	8.6	5.2	49.7	105	37	34	174

$(D_s\text{-}D)/D > 0.10$) were omitted. The remaining data were calibrated for measurement differences between tape and sighted data. The calibrated diameter outside bark (d_{ob}) was computed as follows:

$$d_{ob} = d_s(D/D_s) \qquad [7]$$

Log length and end diameters outside bark (d_{ob}) that corresponded to the Tele-relaskop measurements were entered into standard scaling algorithms used by the USDA Forest Service in the Pacific Northwest Region to compute measured cubic volume inside bark using Smalian's formula.

Standing Tree Estimate of Form Class

Girard form class is needed to apply the Behre profile model as implemented in the Pacific Northwest Region. Traditionally, form class for standing trees had been ocularly estimated on the three NF's from which validation data were collected. However, it was suspected that a prediction model, using d.b.h. and total height, might be a more accurate estimator of Girard form class. A prediction model for form class was fit using the independent validation data.

Two predictor variables were available to estimate Girard form class; diameter at breast height (D) and total tree height (H). The regression model used Girard form class (F) as the dependent variable, which has units of percentage of d.b.h. Several transformations of the predictor variables were explored: D, H, $1/D$, $1/H$, $\log(D)$, and $\log(H)$. $1/D$ and $1/H$ accounted for the most variation. The selected prediction equation was:

$$F = c_8 + c_9(1/H) + c_{10}(1/D) \qquad [8]$$

where c_8, c_9, and c_{10} are regression coefficients. This model was fit using step-wise regression, and estimated model parameters are given in table 5.

Comparison of Profile Models

The Behre model predicted diameter inside bark with mean errors ranging from −0.61 to 0.72 inch (table 6) depending upon species and national forest. The mean error for the Max and Burkhart model ranged between −1.43 and 0.33 inches. However, this difference in bias could be caused by applying a regional model (i.e., the Max and Burkhart model with parameter estimates in tables 1 and 2) to a subregional data set. When the Max and Burkhart model was fit with the same data used for the form class model for the Behre profile model, the Max and Burkhart model was less biased (table 6) than the Behre model.

Table 7 compares the Max and Burkhart and second-stage models with the Behre model. However, predictions of cubic volume, rather than log end diameters, are used as the evaluation criterion. The predicted diameters at log ends are used to compute cubic volume using algorithms implemented by the Forest Service. The Max and Burkhart model had mean errors between −18.7% to 4.2%, with one-third of the mean errors within 2% of zero. The Behre model had less mean error in volume prediction; mean error ranged from −14.1% to 5.2%, but more than half the mean errors were within 2.0% of zero. The Behre profile model uses a prediction of Girard form class, which was fit with the same data that is used to evaluate differences in volume estimates. Therefore, the Behre model is more accurate in this evaluation than can be normally expected, which confounds the assessment of this model.

For Douglas-fir on the Siuslaw NF, both profile models had relatively large mean errors in volume predictions, with the Max and Burkhart model overestimating volume by 10.9%, and the Behre model overestimating volume by 14.1% (table 7). However, both models had mean errors for Douglas-fir from the Mt. Hood and Deschutes NF's that were much closer to zero. For

Table 5.—Residual error summary, regression statistics, and regression coefficients for the Girard form class model (equation [8]). Validation data were used to estimate parameters.

National forest species	Residual error[a]	Regression statistics		Regression coefficients[b]		
	t-stat.	RMSE	R^2	c_8	c_9	c_{10}
Deschutes						
Grand fir	0.90	4.83	0.68	96.83149	-14.51006	0.0
Shasta fir	1.21	5.27	0.61	97.84278	-14.21396	0.0
Lodgepole pine	1.69	6.63	0.36	89.02808	-14.58098	1.45277
Ponderosa pine	0.77	7.27	0.58	91.02634	-12.53353	0.0
Douglas-fir	0.63	3.83	0.44	87.79993	-21.99759	2.06777
Western hemlock	1.40	—	—	73.85463	0.0	0.0
Mt. Hood						
Grand fir	0.60	—	—	82.77240	0.0	0.0
Pacific silver fir	0.70	5.98	0.52	93.54000	-13.43572	1.04066
Western larch	0.95	—	—	78.79044	0.0	0.0
Lodgepole pine	0.81	5.44	0.44	93.19660	-8.69257	0.0
Ponderosa pine	0.59	5.22	0.60	88.30125	-10.83632	0.0
Douglas-fir	0.95	5.47	0.18	84.77447	-11.83024	1.11387
Western hemlock	0.49	5.83	0.33	94.44665	-8.96235	0.0
Siuslaw						
Douglas-fir	0.75	15.00	0.30	105.43757	-61.19910	4.55030
Western hemlock	0.95	—	—	83.50996	0.0	0.0

[a]*The range of the mean residual errors was 0.000127 to -0.000177 except for Douglas-fir in the Siuslaw with a residual error of 0.83235%.*

[b]*The dimensions of regression coefficients are:* c_8, *dimensionless constant;* c_9, *(*ft^{-1}*), and* c_{10} *(*in^{-1}*).*

Table 6.—Comparison of Max and Burkhart and Behre d_{ib} estimates with validation data.

National forest	No. of trees	No. of sections	Max and Burkhart and second-stage models fit to regional data		Behre model fit to validation model		Max and Burkhart and second-stage models fit to validation data	
			Mean error (inches)	t-stat.	Mean error (inches)	t-stat.	Mean error (inches)	t-stat.
Deschutes								
Grand fir	28	86	-0.62	-7.046	-0.35	-3.216	-0.16[a]	-1.612
Shasta fir	18	58	-0.35	-2.714	-0.06	-0.439	0.27[a]	1.543
Lodgepole pine	76	202	-0.25	-4.175	0.33	3.060	-0.17[a]	-2.685
Ponderosa pine	89	261	-1.24	-15.295	-0.05	-0.568	-0.25[a]	-2.540
Douglas-fir	35	120	-0.79	-7.734	-0.20	-1.714	-0.13[a]	-1.580
Western larch	14	54	-1.43	-9.990	-0.61	-2.355	-0.17[a]	-0.837
Mt. Hood								
Shasta fir	70	263	0.25	2.596	0.45	4.951	-0.08	-0.987
Pacific silver fir	77	316	0.27	4.709	0.19	2.930	-0.06	-0.891
Western larch	28	133	-0.38	-3.477	0.04	0.414	0.01	0.081
Lodgepole pine	44	166	-0.17	-2.701	0.48	3.785	-0.02	-0.230
Ponderosa pine	78	368	-1.04	-12.166	0.72	7.738	-0.31[a]	-3.200
Douglas-fir	215	1,139	0.33	5.992	0.40	6.814	-0.08	-1.552
Western hemlock	139	643	-0.34	-5.151	-0.02	-0.334	-0.04	-0.551
Siuslaw								
Douglas-fir	177	1,770	-1.07	-21.619	-0.59	-9.175	0.04	0.781
Western hemlock	86	859	-1.11	-21.003	-0.45	-7.368	0.01	0.331

[a]*No second-stage model found.*

Table 7.—Comparison of average percentage differences in merchantable board foot volume estimates for the Max and Burkhart and second-stage models, and the Behre model, using independent validation data. Negative values indicate overestimates of volume.

			Max and Burkhart model vs. true cubic volume				Behre model vs. true cubic volume				Max and Burkhart model vs. Behre model			
				Cubic foot volume				Cubic foot volume				Cubic foot volume		
	No. of trees	Mean cubic vol. per tree	Percent error	Mean error	Residual SD	*t*-stat.	Percent error	Mean error	Residual SD	*t*-stat.	Percent error	Mean error	Residual SD	*t*-stat.
Deschutes NF														
Grand fir	28	36.3	-6.7	-2.5	3.71	-3.49	-3.6	-1.3	3.78	-1.85	-6.5	-1.1	0.68	-8.72
California red fir	18	50.8	-2.4	-1.2	5.80	-0.91	-1.2	-0.6	5.87	-0.43	-2.4	-0.6	1.47	-1.88
Lodgepole pine	76	19.1	-3.7	-0.7	2.71	-2.28	1.1	0.2	2.56	0.72	-3.8	-0.9	1.23	-6.52
Ponderosa pine	89	52.8	-12.6	-6.6	11.75	-5.33	-0.2	-0.1	9.70	-0.08	-12.6	-6.6	7.57	-8.18
Douglas-fir	35	48.0	-8.0	-3.8	6.86	-3.32	-1.6	-0.8	7.02	-0.66	-7.9	-3.1	2.12	-8.57
Western larch	14	67.8	-18.7	-12.6	25.89	-1.83	-13.6	-9.2	37.07	-0.93	-16.4	-3.5	12.60	-1.02
Mt. Hood NF														
Shasta fir	70	85.8	-1.8	1.6	14.33	0.91	2.2	1.9	13.71	1.16	1.9	-0.3	4.20	-0.68
Pacific silver fir	77	83.6	3.6	3.0	11.02	2.39	2.0	1.7	11.85	1.25	3.7	1.3	4.38	2.62
Western larch	28	94.9	-1.7	-1.6	12.80	-0.68	1.6	1.5	13.09	0.61	-1.8	-3.2	4.07	-4.13
Lodgepole pine	44	30.0	-1.9	-0.6	3.34	-1.15	1.4	0.4	3.64	0.78	-2.0	-1.0	1.70	-3.94
Ponderosa pine	78	178.9	-9.7	-17.3	33.93	-4.50	5.2	9.3	32.13	2.55	-10.2	-26.5	24.73	-9.48
Douglas-fir	215	262.8	-4.2	11.1	56.37	2.89	2.0	5.3	52.32	1.48	4.3	5.8	27.04	3.15
Western hemlock	139	169.2	-1.3	-2.2	25.27	-1.04	0.3	0.6	23.93	0.29	-1.3	-2.8	6.93	-4.76
Siuslaw NF														
Douglas-fir	177	292.4	-10.9	-31.8	64.32	-6.58	-14.1	-41.3	90.46	-6.07	-9.5	9.5	40.18	3.14
Western hemlock	86	117.9	-13.8	-16.3	30.89	-4.88	-2.3	-2.7	27.80	-0.90	-13.5	-13.6	18.12	-6.94

western larch from the Deschutes NF, both models overestimated volume by an average of over 13% (table 7). However, both models had mean errors within 2% for western larch from the Mt. Hood NF. Max and Burkhart model overestimated volume of western hemlock from the Siuslaw NF by an average of 13.8%, whereas the Behre model was much less biased. However, bias of the Max and Burkhart model was much less for western hemlock on the Mt. Hood NF, with a mean error of −1.3%. The Max and Burkhart model tended to overestimate volume of ponderosa pine by an average of 12.6% on the Deschutes NF, and 9.7% on the Mt. Hood NF. The Behre model had less bias for ponderosa pine volume (table 7). All other differences in mean error of volume estimates (table 7) were smaller for both models.

The Max and Burkhart model is not expected to produce merchantable volume estimates that are radically different from the Behre model, which is currently used in practice. The differences in volume predictions between models are compared in table 7. On the average, differences between models were less than 10% for all but four cases. The greatest differences between models were in predictions of merchantable volume for western larch (16.4%) on the Deschutes NF, although differences were much less on the Mt. Hood NF (1.8%). There were also differences for ponderosa pine; volume predictions from the Max and Burkhart model were larger than those from the Behre model by an average of 10.2% for the Mt. Hood NF, and 12.6% for the Deschutes NF. The Max and Burkhart model also produced larger volume estimates (13.5%) for western hemlock on the Siuslaw NF, although the two models were in much closer agreement for western hemlock on the Mt. Hood NF (1.3%). Therefore, in most cases, the Max and Burkhart model can be expected to produce volume estimates similar to those from the currently used Behre model, without the accurate standing tree estimates or predictions of Girard form class that are needed by the Behre model.

Discussion

One of the purposes of the second-stage model is to adjust for bias in predicted diameter from the Max and Burkhart model that is manifested during the retransformation of estimated $(d_{ib}/D)^2$ to estimates of d_{ib}. The transformation (d_{ib}/D) is used to stabilize variance, which is required when using traditional (i.e., unweighted) regression techniques. The squaring of this transformation [i.e., $(d_{ib}/D)^2$] increases precision of the volume estimates and reduces the nonlinearity of the profile model while continuing to stabilize variance. Diameter (d_{ib}) could be used directly as the response variable in the regression model, and this would avoid overall retransformation bias. However, the variance of the residuals would be dependent on diameter, d.b.h., and measurement height, which violates the assumptions in the usual ordinary least squares linear regression model. Also, more weight would be placed on minimizing prediction errors for large trees, and diameters near the base of the tree, because the magnitude of the errors would be larger. Conceivably, this could produce models that predict negative upper stem diameters, or poorly represent smaller trees. The (d_{ib}/D) or $(d_{ib}/D)^2$ transformations put smaller on a more equal metric with the larger trees and lower stem measurements. The (d_{ib}/D) transformation could have been used in place of the $(d_{ib}/D)^2$ transformation, but this would have introduced additional nonlinearities into the regression model, and would have imposed additional costs. Weighted regression could also be used to more directly treat the

heteroscedasticity of d_{ib}. Carrol and Ruppert (1988) provide a recent summary of the relevent literature. However, elimination of the correlations of residuals with D and D/H (figs. 3 and 4) likely requires more than weighted regression.

The second-stage models correct for mean errors of 0.06 to 0.24 inch in diameter predictions; these systematic biases, although relatively small, can translate into substantial volume estimation errors when applied to a large geographic region. It is difficult to defend overestimation of merchantable volume, especially when the bias is easy to correct empirically on the average. However, the bias for any one timber sale can be high, even when using a model that is unbiased for a large region. There is naturally high variability in stem form at the local level, and there is no inexpensive way to measure this variablity so that every local estimate is unbiased. Even so, the second-stage model, which contains a covariate of tree form (i.e., D/H), can improve local estimates (Czaplewski et al. 1989).

As precision of the diameter predictions improves, individual diameter estimates from an unbiased model will be closer to their true values. The predictor variables in the second-stage model (i.e., D, D/H, h, h^2) improve precision; the standard deviation of residuals from the second-stage model for all trees in the appendix is 3% less than that from the stem profile models alone.

The risk of using the second-stage model is small. Residuals from the second-stage models have been closely scrutinized, and no problems have been detected. Certain problems, which are possible, although unlikely, can be readily checked when the second-stage models are applied in practice. These checks are simple to implement; e.g., verify that diameter estimates are positive and decrease going up the main stem. Also, both the old volume estimators and new profile models can be used during a trial period; if there are unidentified risks, then they will probably surface during a trial period. Risks of not using the second-stage models are larger. For example, overestimates of merchantable volume for large geographic areas from the unadjusted Max and Burkhart model could distort analyses in forest management plans.

Emphasis has been placed on minimizing bias in predictions of upper stem diameters, even though volume estimation is the final goal. It is common practice in research studies to integrate stem profile models to estimate cubic volume (Martin 1981). However, the Forest Service and many other forest management organizations use log length and log end diameters to scale (i.e., estimate) merchantable log volume. Also, merchantable board foot volume is defined primarily by the upper end diameter estimate, rather than the integrated cubic volume estimate. Even in detailed stem analysis studies, section lengths and end diameters are usually measured to indirectly estimate cubic volume (e.g., Smalian's formula); water displacement methods are seldom used to measure volume directly (Martin 1984). Therefore, integrated cubic volume estimates might not represent the same definition of volume that is used in the field.

Cubic volume estimates from section length and predicted end diameters will differ slightly from cubic volume estimates obtained from integrating the stem profile model. However, it is prudent to maintain consistency in volume estimation for timber sale preparation, log scaling, inventory, and planning. Therefore, the stem profile and second-stage models are used to predict the number of merchantable logs in a tree, and diameters at heights that correspond to log ends. These predictions are entered into existing algorithms to scale merchantable log volume. This increases the data processing load, but internal flexibility and consistency are maintained. Similar procedures have been used for decades in the Pacific Northwest by both forest industry and government land management agencies. Scaled cubic volume from a profile model will be highly correlated with integrated cubic volume from the same model.

Without the second-stage model, the Max and Burkhart stem profile model can readily estimate diameters at given heights, heights to given top diameters, and cubic volume of stem sections. Although this provides considerable convenience, many of these predictions contain bias. However, the second-stage model can produce approximately unbiased estimates of stem diameter. Presumably, this will reduce bias in predictions of merchantable height (at the cost of increased computations), and volume predictions that use diameter predictions from the second-stage model as input variables to scaling algorithms. Therefore, the second-stage model can provide both mensurational and institutional benefits.

The stem profile models in this paper are expected to provide useful predictions of merchantable volume given a variety of merchantability standards and scaling rules. These predictions are expected to be unbiased, when many local sites are averaged together, and when applied to the same population of trees (see appendix) that were used to estimate regression parameters. It is not known how well these models will predict diameters for trees from other populations. The distribution of tree form and size can change over time, especially for second-growth stands, and new stem profile models will be required at some time in the future.

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John Teply originally proposed that emphasis be placed on unbiased predictions of diameter, rather than other estimates that can be produced from a stem profile model. Tim Max suggested substantial improvements in the development and verification of the second-stage model. David Bruce enriched our critical examination of the methods and interpretations.

Literature Cited

Alberta Forest Service. 1987. Evaluation and fitting of tree taper functions for Alberta. Pub. No. Dept. 106. Edmonton, AB: Silva Com. Ltd. 148 p.

Amidon, E. L. 1984. A general taper functional form to predict bole volume for five mixed-conifer species in California. Forest Science. 30: 166-171.

Behre, C. E. 1923. Preliminary notes on studies of tree form. Journal of Forestry. 21: 507-511.

Bruce, D. 1972. Some transformations of the Behre equation of tree form. Forest Science. 18: 164-166.

Cao, V. Q.; Burkhart, H. E.; Max, T. A. 1980. Evaluation of two methods for cubic volume prediction of loblolly pine to any merchantable limit. Forest Science. 26: 71-80.

Carroll, Raymond J.; Ruppert, David. 1988. Transformation and weighting in regression. New York, NY: Chapman and Hall.

Czaplewski, R. L. 1989. Graphical analysis of stem taper in model building. Canadian Journal of Forest Research. 19: 522-524.

Czaplewski, R. L.; Brown, A. S.; Walker, R. C. 1989. Stem profile models in Wyoming, Colorado, and South Dakota. Res. Pap. RM-284. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station. 7 p.

Draper, N. R.; Smith, H. 1981. Applied regression analysis. New York, NY: John Wiley and Sons. 709 p.

Flewelling, J. W.; Pienaar, L. V. 1981. Multiplicative regression with lognormal errors. Forest Science. 27: 281-289.

Gordon, A. 1983. Comparison of compatible polynomial taper equations. New Zealand Journal of Forestry Science. 13: 146-153.

Maguire, D. A.; Hann, D. W. 1987. Equations for predicting sapwood area at crown base in southwestern Oregon Douglas-fir. Canadian Journal of Forest Research. 17: 236-241.

Martin, A. J. 1981. Taper and volume equations for selected Appalachian hardwood species. Res. Pap. NE-490. Broomall, PA: U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station. 22 p.

Martin, A. J. 1984. Testing volume equation accuracy with water displacement techniques. Forest Science. 30: 41-50.

Max, T. A.; Burkhart, H. E. 1976. Segmented polynomial regression applied to taper equations. Forest Science. 22: 283-289.

Sterba, H. 1980. Stem curves--a review of the literature. Forestry Abstracts. 41: 141-145.

Taylor, J. M. G. 1986. The retransformed mean after a fitted power transformation. Journal of American Statistical Association. 81: 114-118.

Appendix

Table A1.—Number of trees for Pacific silver fir.

Region	D.b.h. class (inches)							Total height class (feet)								Total
National Forest	7-11.9	12-13.9	14-14.9	15-16.9	17-19.9	20-22.9	23-48.9	35-59.9	60-69.9	70-79.9	80-84.9	85-99.9	100-114.9	115-170	trees
Northwestern Oregon & Southern Washington															
Gifford Pinchot	2	12	12	35	38	32	20	2	9	47	14	29	36	14	151
Mt. Hood	0	8	3	16	13	10	24	0	1	17	8	19	8	21	74
Willamette	1	4	3	9	18	9	15	0	2	10	8	17	13	9	59
Total	3	24	18	60	69	51	59	2	12	74	30	65	57	44	284

Table A2.—Number of trees for grand fir.

Region	D.b.h. class (inches)							Total height class (feet)								Total
National Forest	7-13.9	14-16.9	17-18.9	19-21.9	22-24.9	25-28.9	29-56	35-59.9	60-69.9	70-79.9	80-84.9	85-94.9	95-109.9	110-205	trees
Central Oregon															
Deschutes	19	25	18	28	29	25	16	22	24	38	11	32	23	10	160
Fremont	3	19	13	24	27	48	35	43	28	37	10	34	15	2	169
Winema	43	35	14	16	24	19	32	48	21	29	9	21	22	33	183
Total	65	79	45	68	80	92	83	113	73	104	30	87	60	45	512
Eastern Oregon															
Malheur	10	32	34	40	26	20	14	45	34	34	18	22	14	9	176
Ochoco	18	23	20	24	20	23	10	29	26	21	11	27	17	7	138
Umatilla	48	33	29	33	21	23	22	29	30	30	21	40	31	28	209
Wallowa-Whitman	20	15	17	22	22	29	42	28	20	20	12	19	40	28	167
Total	96	103	100	119	89	95	88	131	110	105	62	108	102	72	690
Northwestern Oregon & Southern Washington															
Mt. Hood	24	1	0	0	0	0	0	7	7	4	5	2	0	0	25
Southwestern Oregon															
Rogue River	30	32	12	29	29	38	50	36	29	17	15	30	30	63	220
Grand Total	215	215	157	216	198	225	221	287	219	230	112	227	192	180	1,447

Table A3.—Number of trees for Shasta fir.

Region	D.b.h. class (inches)							Total height class (feet)							Total
National Forest	11-14.9	15-17.9	18-20.9	21-24.9	25-29.9	30-34.9	35-67	30-54.9	55-64.9	65-84.9	85-99.9	100-114.9	115-129.9	130-175	trees
Central Oregon															
Winema	25	31	15	15	8	0	0	39	21	25	6	3	0	0	94
Southwestern Oregon															
Rogue River	7	16	20	22	36	34	44	11	9	23	32	34	39	31	179
Grand Total	32	47	35	37	44	34	44	50	30	48	38	37	39	31	273

Table A4.—Number of trees for western larch.

Region	D.b.h. class (inches)							Total height class (feet)							Total
National Forest	9-13.9	14-15.9	16-17.9	18-20.9	21-23.9	24-27.9	28-47	40-74.9	75-84.9	85-89.9	90-94.9	95-104.9	105-114.9	115-175	trees
Eastern Oregon															
Umatilla	29	33	34	47	19	14	9	53	36	25	21	27	8	15	185
Wallowa-Whitman	14	11	22	25	32	31	51	19	21	22	26	30	39	29	186
Total	43	44	56	72	51	45	60	72	57	47	47	57	47	44	371

Table A5.—Number of trees for lodgepole pine.

Region National Forest	D.b.h. class (inches)							Total height class (feet)						Total trees
	6-8.9	9-9.9	10-10.9	11-11.9	12-13.9	14-15.9	16-27	40-49.9	50-54.9	55-59.9	60-64.9	65-74.9	75-105	
Central Oregon														
Deschutes	0	5	9	10	30	21	43	18	13	21	19	32	15	118
Fremont	6	19	27	29	43	34	35	51	28	33	28	42	11	193
Winema	2	23	36	33	43	14	12	35	42	31	28	19	8	163
Total	8	47	72	72	116	69	90	104	83	85	75	93	34	474
Eastern Oregon														
Umatilla	43	26	30	12	18	12	8	20	37	39	11	26	16	149
Northwestern Oregon & Southern Washington														
Gifford Pinchot	0	2	4	1	14	14	141	4	4	5	4	20	139	176
Mt. Hood	0	0	0	4	17	21	135	2	2	7	2	15	149	177
Siuslaw	0	0	0	0	0	0	3	0	0	0	0	0	3	3
Willamette	0	1	4	3	14	10	140	2	2	2	6	9	151	172
Total	0	3	8	8	45	45	419	8	8	14	12	44	442	528
Grand Total	51	76	110	92	179	126	517	132	128	138	98	163	492	1,151

Table A6.—Number of trees for ponderosa pine.

Region National Forest	D.b.h. class (inches)							Total height class (feet)							Total trees
	9-16.9	17-20.9	21-22.9	23-25.9	26-29.9	30-33.9	34-61	40-64.9	65-74.9	75-79.9	80-89.9	90-99.9	100-114.9	115-160	
Central Oregon															
Deschutes	30	43	24	35	45	35	46	48	31	22	32	31	53	41	258
Fremont	35	39	21	16	35	10	16	61	47	11	21	19	10	3	172
Winema	22	38	28	32	37	17	18	27	28	23	39	33	29	13	192
Total	87	120	73	83	117	62	80	136	106	56	92	83	92	57	622
Eastern Oregon															
Malheur	34	29	18	35	27	26	33	42	32	20	30	38	31	9	202
Ochoco	17	36	19	38	20	33	22	21	22	24	35	32	29	22	185
Umatilla	26	34	29	49	66	52	31	45	34	22	62	49	45	30	287
Wallowa-Whitman	36	45	23	37	41	36	36	64	52	22	41	40	22	13	254
Total	113	144	89	159	154	147	122	172	140	88	168	159	127	74	928
Northwestern Oregon & Southern Washington															
Mt. Hood	10	16	22	21	31	41	44	18	22	10	16	25	50	44	185
Grand Total	210	280	184	263	302	250	246	326	268	154	276	267	269	175	1,735

Table A7.—Number of trees for Douglas-fir.

Region National Forest	D.b.h. class (inches)							Total height class (feet)							Total trees
	7-14.9	15-19.9	20-23.9	24-27.9	28-32.9	33-39.9	40-77	40-74.9	75-89.9	90-104.9	105-119.9	120-134.9	135-159.9	160-250	
Central Oregon															
Deschutes	12	19	30	35	30	24	10	20	29	48	38	13	12	0	160
Eastern Oregon															
Ochoco	21	40	36	24	29	23	6	51	44	41	36	6	1	0	179
Umatilla	45	51	55	28	26	6	2	101	49	31	18	8	5	1	213
Total	66	91	91	52	55	29	8	152	93	72	54	14	6	1	392
Northwestern Oregon & Southern Washington															
Gifford Pinchot	27	29	25	19	23	28	31	15	37	28	28	30	30	14	182
Mt. Hood	80	63	48	55	59	54	56	71	81	79	70	38	43	33	415
Siuslaw	28	49	37	48	58	61	43	7	7	16	42	54	92	106	324
Willamette	12	13	31	26	28	29	31	9	15	17	24	27	32	46	170
Total	147	154	141	148	168	172	161	102	140	140	164	149	197	199	1,091
Southwestern Oregon															
Siskiyou	10	14	20	20	23	40	53	6	13	19	22	29	47	44	180
Grand Total	235	278	282	255	276	265	232	280	275	279	278	205	262	244	1,823

Table A8.—Number of trees for western hemlock.

Region National Forest	D.b.h. class (inches)							Total height class (feet)							Total trees
	9-13.9	14-16.9	17-18.9	19-22.9	23-26.9	27-33.9	34-57	40-74.9	75-89.9	90-99.9	100-109.9	110-124.9	125-139.9	140-195	
Northwestern Oregon & Southern Washington															
Gifford Pinchot	21	23	22	31	31	19	30	37	29	22	29	26	16	18	177
Mt. Hood	21	34	23	22	16	32	31	28	35	25	20	19	21	31	179
Siuslaw	0	0	0	0	0	3	0	0	0	0	0	0	1	2	3
Willamette	22	19	16	43	30	25	17	21	30	20	22	30	26	23	172
Grand Total	64	76	61	96	77	79	78	86	94	67	71	75	64	74	531

Czaplewski, Raymond L.; Brown, Amy S.; Guenther, Dale G. 1989. Estimating merchantable tree volume in Oregon and Washington using stem profile models. Res. Pap. RM-286. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station. 15 p.

The profile model of Max and Burkhart was fit to eight tree species in the Pacific Northwest Region (Oregon and Washington) of the Forest Service. Most estimates of merchantable volume had an average error less than 10% when applied to independent test data for three national forests.

Keywords: Taper equations, merchantable volume estimates, bias correction, *Abies amabilis*, *Abies grandis*, *Abies magnifica*, *Larix occidentalis*, *Pinus contorta*, *Pinus ponderosa*, *Pseudotsuga menziesii*, *Tsuga heterophylla*

U.S. Department of Agriculture
Forest Service

Rocky Mountain Forest and Range Experiment Station

Rocky Mountains

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

Southwest

RESEARCH FOCUS

Research programs at the Rocky Mountain Station are coordinated with area universities and with other institutions. Many studies are conducted on a cooperative basis to accelerate solutions to problems involving range, water, wildlife and fish habitat, human and community development, timber, recreation, protection, and multiresource evaluation.

Great Plains

RESEARCH LOCATIONS

Research Work Units of the Rocky Mountain Station are operated in cooperation with universities in the following cities:

Albuquerque, New Mexico
Flagstaff, Arizona
Fort Collins, Colorado*
Laramie, Wyoming
Lincoln, Nebraska
Rapid City, South Dakota
Tempe, Arizona

*Station Headquarters: 240 W. Prospect Rd., Fort Collins, CO 80526